A note on demi Dunford-Pettis operators

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Let X and Y be Banach spaces and X', Y' be norm dual of X, Y, respectively. A linear bounded operator $T : X \to Y$ is called a Dunford-Pettis operator if $x_n \to 0$ weakly in X implies $\| Tx_n \| \to 0$ in Y. Many authors are interested in this subject. Adjoint operator of $T : X \to Y$ is defined by $T' : Y' \to X'$, $(T'f)(x) = f(Tx)$ for every $f \in Y', x \in X$. An operator $T : X \to X$ is said to be demi Dunford-Pettis operator if, for every sequence (x_n in X such that $x_n \to 0$ weakly in X with $\|x_n - Tx_n\| \to 0$,then $\|x_n\| \to 0$ in X.Every Dunford-Pettis operator is a demi Dunford-Pettis operator. An operator $T : X \to Y$ is called a weakly Dunford-Pettis operator if, for every sequence (x_n) in X with $x_n \to 0$ weakly in X and (f_n) in Y' with $f_n \to 0$ weakly in Y' , then $f_n(Tx_n) \to 0$ as $n \to \infty$. We define weakly demi Dunford-Pettis operator that it is different from defined by J. Riberio and F. Santos. We show that the results obtained by J. Riberio and F. Santos are also true by using our definition.Also, we investigate the results on Banach lattices.

Theorem 1 ([2])**.** *Every weakly Dunford-Pettis operator is a weakly demi Dunford-Pettis operator.*

References

[1] C.D. Aliprantis, O. Burkinshaw *Positive Operators*, Springer,2006. On a new superposition principle for optimization problems. *Russian Mathematical Surveys*, 42(3) : 43–54, 1987.

[2] J. Riberio, F. Santos, *Demi weakly Dunford-Pettis on Banach spaces, arXiv, 2603.08922v*, 9 Mar 2026.