Hamiltonian Lax type flows related to the centrally extended Lie algebra of matrix super-integro-differential operators with two anticommuting variables and their rational factorization

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In the paper [3] the Lie algebra of matrix super-integro-differential operators with one anticommuting variable, which possesses an invariant with respect to the corresponding commutator scalar product and standard splitting into a direct sum of its Lie subalgebras, has been considered. It has been applied to construct Lax integrable hierarchies of nonlinear dynamical systems on supermatrix-valued supermanifolds of one commuting and one anticommuting independent variables as reductions of the hierarchy of Hamiltonian Lax type flows in its dual space, generated by the associated $\mathcal{R}$-deformed Lie–Poisson bracket and Casimir functionals, upon coadjoint orbits. The mentioned hierarchy and set of hierarchies of its additional Hamiltonian homogeneous symmetries for certain type coadjoint orbits have been used to obtain the nonlinear dynamical systems on supermatrix-valued supermanifolds of two commuting and one anticommuting independent variables, which admit triple Lax type linearizations.

Basing on the Lie-algebraic Adler–Kostant–Symes scheme and $\mathcal{R}$-operator approach, in [4] one obtains the hierarchy of Hamiltonian Lax type flows in the dual space to the centrally extended by the Maurer–Cartan two-cocycle parameterized Lie algebra of matrix super-integro-differential operators with one anticommuting variable. On the corresponding coadjoint orbits it is reduced to some Lax integrable hierarchies of nonlinear dynamical systems on supermatrix-valued supermanifolds of two commuting and one anticommuting independent variables. There the rational factorization (see, for example, [1, 5, 2, 6]) of this hierarchy has been also investigated.

In the report, that is proposed, one introduces the Lie algebra of matrix super-integro-differential operators with two anticommuting variables, endowed with an invariant scalar product, which admits a non-standard splitting into a direct sum of its Lie subalgebras and develops a new Lie-algebraic method for constructing Lax integrable hierarchies of nonlinear dynamical systems on supermatrix-valued supermanifolds of two commuting and two anticommuting independent variables by means of the hierarchy of Hamiltonian Lax type flows in the space dual to the central extension of its parameterized version.

On the introduced Lie algebra the non-standard splitting generates some $\mathcal{R}$-operator which is used to obtain a new $\mathcal{R}$-deformed commutator on the corresponding central extension. The mentioned Hamiltonian Lax type flows are determined by the Lie–Poisson bracket associated with $\mathcal{R}$-deformed commutator on the central extension and Casimir functionals as Hamiltonians. The rational factorization problem for these Hamiltonian flows is also studied.

By means of the Bäcklund mapping given by the rational factorization, one shows that the system of Hamiltonian Lax type flows for a pair of different elements of the parameterized Lie algebra of matrix super-integro-differential operators with two anticommuting variables, which are related by the generalized gauge transformation, is equivalent to the system of two evolution equations for the polynomials in two superderivatives, rationally factorizing these elements, and proves the Hamiltonicity of the latter system. A new method for finding the Poisson operator that generates the corresponding Hamiltonian representation in an explicit form is proposed.

The rational factorization method substantiated in this report can be applied to construct new integrable hierarchies of nonlinear dynamical systems on supermatrix-valued supermanifolds of two commuting and two anticommuting independent variables and infinite sequences of their conservation law gradients.

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