Some classes of surfaces with Grassmann image of constant curvature in the Minkowski space

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The range of questions of differential geometry of surfaces in multidimensional spaces is significantly expanded if we use the properties of their Grassmann images. This work presents a study of non-isotropic minimal surfaces and surfaces with flat normal connection in the Minkowski space ${}^1R_4$ that have a non-degenerate Grassmann image of constant curvature $\overline{K}$.

Let V^2 be a two-dimensional surface of the class C^k, $k \geq 1$ in the Minkowski space ${}^1R_4$ with metric form $ds^2 = -dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$. A surface in the space ${}^1R_4$ is called *time-like (space-like)* if its tangent plane at every point is time-like (space-like).

The number $H^k = \frac{g^{ij}L_{ij}^k}{2}$ is called the mean curvature of the surface in the direction of the normal vector $\bar{\xi}_k$, and the vector $H = H^1\bar{\xi}_1 + H^2\bar{\xi}_2$ is the mean curvature vector. The surfaces of the Minkowski space with zero mean curvature vector will be called minimal surfaces as in Euclidean space.

Theorem 1. *A time-like minimal surface has a non-degenerate Grassmann image of constant curvature $\overline{K} = 1$ if and only if it is a hypersurface of a three-dimensional subspace of the Minkowski space. Time-like minimal surfaces with a non-degenerate Grassmann image of constant curvature $\overline{K} = 1$ exist.*

Theorem 2. *In the Minkowski space, there are no time-like minimal surfaces with a non-degenerate Grassmann image of constant curvature other than* 1*.*

Theorem 3. *A space-like minimal surface of the Minkowski space with a non-degenerate Grassmann image of the constant curvature $\overline{K}$ can only be a hypersurface of a three-dimensional subspace of the Minkowski space (i.e. can only have curvature $\overline{K} = -1$), and its Gauss curvature should be non-constant. In the Minkowski space there exist space-like minimal surfaces with a non-degenerate Grassmann image of the constant curvature $\overline{K}$.*

We have described [1] all two-dimensional non-isotropic surfaces with flat normal connection in the Minkowski space, whose non-degenerate Grassmann image has constant curvature. The following theorem has been proven.

Theorem 4. *For every $k \in [0, 1]$, in the Minkowski space, there exists a two-dimensional time-like surface V^2 of the class C^n, $n \geq 3$, with flat normal connection the non-degenerate Grassmann image of which has the constant curvature k. For every $k \in (-\infty, -1]$, in the Minkowski space, there exists a two-dimensional space-like surface V^2 of the class C^n, $n \geq 3$, with flat normal connection the non-degenerate space-like Grassmann image that has the constant curvature k. For every $k \in [0, \infty)$, in the Minkowski space, there exists a two-dimensional space-like surface V^2 of the class C^n, $n \geq 3$, with flat normal connection the non-degenerate time-like Grassmann image of which has the constant curvature k.*

References

[1] P. Stegantseva and M. Grechneva. Two-dimensional nonisotropic surfaces with flat normal connection and a nondegenerate Grassmann image of constant curvature in the Minkowski space. *Ukrains'kyi Matematychnyi Zhurnal*, 76(4):533–551, 2024.