On loops that satisfy $x \cdot (x \cdot yx)z = (x \cdot xy) \cdot xz$

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An LTWC is a loop that satisfies $x \cdot (x \cdot yx)z = (x \cdot xy) \cdot xz$. LTWC loops are proved to be power associative and left conjugacy closed (LCC). An LCC loop is LTWC if and only if $x(x \cdot yx) = (x \cdot xy)x$. Connections to left Bol loops, left Cheban loops and loops satisfying $(xy \cdot x) \cdot xz = x \cdot (yx \cdot x)z$ (LWPC) are also considered.

This paper is mainly concerned with loops fulfilling the law

$$x \cdot (x \cdot yx)z = (x \cdot xy) \cdot xz \qquad \text{(LTWC)}$$

This law was introduced in [2], where it is dubbed (Q_{12}). It may be regarded as a 'twist' of the law

$$(xy \cdot x) \cdot xz = x \cdot (yx \cdot x)z \qquad \text{(LWPC)}$$

that was first formulated by Phillips in [4] and investigated in [3, 1].

$$y(xz \cdot x) \cdot x = yx \cdot (zx \cdot x) \qquad \text{(RTWC)}$$

$$zx \cdot (x \cdot yx) = z(x \cdot xy) \cdot x \qquad \text{(RWPC)}$$

$$yx \cdot xz = (y \cdot zx)x \qquad \text{(RCh)}$$

Lemma 1. *Let x and y be elements of a loop Q.*
(i) If Q fulfills (LTWC) or (RWPC), then $x(x \cdot yx) = (x \cdot xy)x$, i.e. $L_x^2 R_x = R_x L_x^2$.
(ii) If Q fulfills (RTWC) or (LWPC), then $(xy \cdot x)x = x(yx \cdot x)$, i.e. $R_x^2 L_x = L_x R_x^2$.
(iii) If Q is left or right Cheban, then $x \cdot xy = yx \cdot x$, i.e. $L_x^2 = R_x^2$.

Theorem 2. *Let Q be a loop. Then*

(i): *Q fulfills (LTWC) if and only if Q is an LCC loop in which $x(x \cdot yx) = (x \cdot xy)x$ for all $x, y \in Q$; and*

(ii): *Q fulfills (LWPC) if and only if Q is an LCC loop in which $x(yx \cdot x) = (xy \cdot x)x$ for all $x, y \in Q$.*

Theorem 3. *Every LTWC loop is power associative.*

Theorem 4.

(1) *If an LWPC-loop is a left Bol loop, then it is an LTWC-loop.*
(2) *A left Bol loop is an LTWC-loop if and only if it is a left Burn loop.*

References

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