Zero-divisors in commutative rings and their graph-theoretic applications

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Zero-divisors play an important role in the study of commutative rings, since they reveal essential information about the internal structure of a ring, its ideals, and its decomposition properties. In this talk, we study zero-divisors in commutative rings through the language of graph theory. More precisely, to a commutative ring R with identity, one associates the zero-divisor graph $\Gamma(R)$, whose vertices are the nonzero zero-divisors of R, and where two distinct vertices x and y are adjacent whenever $xy = 0$. This construction translates an algebraic relation into a combinatorial object, allowing ring-theoretic properties to be investigated by means of graph-theoretic tools.

The aim of the talk is to present the main algebraic and graph-theoretic ideas behind this construction. We first recall the basic notions of commutative rings, zero-divisors, nilpotent elements, idempotent elements, ideals, and annihilators. We then explain how the structure of zero-divisors influences the shape of the associated graph. In particular, we discuss graph invariants such as connectedness, diameter, girth, clique number, independence number, chromatic number, and planarity. These invariants provide useful information about the way zero-divisors interact inside the ring.

Special attention is given to finite commutative rings, where the zero-divisor graph gives a concrete and visual representation of algebraic phenomena. Examples such as $\mathbb{Z}/n\mathbb{Z}$, especially for composite integers n, illustrate how the factorization of n affects the set of zero-divisors and the corresponding graph. The talk also highlights the connection between zero-divisor graphs and ideal structures, showing how adjacency relations may reflect annihilation, ideal intersection, and decomposition behavior.

Finally, we indicate some possible applications and perspectives of zero-divisor graphs in computational algebra, coding theory, cryptography, and network-type models. The general purpose is to show that zero-divisors, far from being merely algebraic obstacles, can serve as a bridge between commutative algebra and graph theory, offering both structural insight and useful combinatorial interpretations.