Introducing the Topological Stability Index

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We introduce the *Topological Stability Index* (TSI), a variance-based scalar measure for persistence barcodes that quantifies the dispersion of persistence lifetimes. Unlike persistent entropy, which depends only on normalized weights, the TSI captures absolute variability and is sensitive to heterogeneous feature scales. We establish fundamental properties of the TSI, including its scaling behavior, invariance under lifetime translation, and explicit update formulas under insertion and deletion of bars. We also consider a complementary first-moment-type quantity, the *Topological Signal Index* (TSigI), which captures the typical scale of persistence lifetimes and provides additional interpretability alongside the TSI. We further introduce a normalized version, *cv*TSI, which is scale invariant and admits an explicit algebraic relation to the Rényi entropy of order two. In particular, *cv*TSI is an affine function of the collision probability $\sum_i p_i^2$, and therefore a monotone reparametrization of the Rényi entropy, providing a direct link between variance-based and entropy-based summaries in topological data analysis. Numerical experiments on synthetic data and stochastic time series demonstrate that the TSI captures structural variability complementary to entropy: it is relatively insensitive to deterministic trends, while responding strongly to stochastic fluctuations and variations in persistence magnitude.

References