# Problem with integral condition for homogeneous system of partial differential equations of fourth order in Sobolev space

**Grzegorz Kuduk**
(Faculty of Mathematics and Natural Sciences University of Rzeszow, Graduate of University of Rzeszow)
*E-mail:* gkuduk@onet.eu

**Michail Symotyuk**
(Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, National Academy of Sciences of Ukraine, Lviv)
*E-mail:* quaternion@ukr.net

In the strip $\Omega(T) : \{(t,x) \in \mathbb{R}^{n+1} : t \in (0,T), x \in \mathbb{R}^n\}$ we consider nonlocal problem with integral conditions

$$L\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)\overrightarrow{U}(t,x) = \frac{\partial^4 \overrightarrow{U}(t,x)}{\partial t^4} + \sum_{i=1}^{4} A_i \frac{\partial^4 \overrightarrow{U}(t,x)}{\partial t^{4-i}\partial x^i} = \overrightarrow{0}, \tag{1}$$

$$\int_{T_1}^{T_2} t^k \overrightarrow{U}(t,x)dt = \overrightarrow{\varphi}_j(x), \quad k = \{0,1,2,3\}, \tag{2}$$

where $\overrightarrow{U}(t,x) = col\left(U^1(t,x), ..., U^4(t,x)\right)$, $\overrightarrow{\varphi}_j(x) = col\left(\varphi_j^1, ..., \varphi_j^4\right)$, $j = \{1,2,3,4\}$, $A_i$, are squert matrix $4 \times 4$,

$$\int_0^T t^k \overrightarrow{U}(t,x)dt = col\left(\int_0^T t^k U_1(t,x)dt, ..., \int_0^T t^k U_4(t,x)dt\right).$$

Main determinand of the system (1) in the form $\Delta(\xi)$. Let $H_\alpha, \alpha > 0$ be a Sobolew space with the finite norm [1].

$$\|\varphi(x); H_\alpha\| = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{+\infty} (1+|\xi|)^{2\alpha} |\widetilde{\varphi}(\xi)|^2 d\xi} < \infty$$

where $\widetilde{\varphi}(\xi)$ is a Fourier transformation of the funktion $\varphi(x)$. $\overline{H}_\alpha$ is a vector space of the funktion $\varphi(x)$, with norm

$$\|\varphi(x), \overline{H}_\alpha\| = \max \|\varphi^j(x); H_\alpha\|$$

**Theorem**. Let conditions occur, let $\Delta(\xi) \neq 0$ for everyone $\xi \in \mathbb{R} \setminus \{0\}$. If $\overrightarrow{\varphi}_j \in \overline{H}^4_{\alpha_1}$, $\alpha_1 \geqslant \alpha_2 + 4\left(C_4^4 + 1\right)$, $\alpha_2 > 1$, $j = \{1, ..., 4\}$, then the space $C^4(0,T), \overline{H}^4_\alpha)$ exists and unique solution $\overrightarrow{U}(t,x)$ of the problem (1)-(2), which still is depending on vector-function $\overrightarrow{\varphi}_j(x)$, $j = \{1, ..., 4\}$. Solution is the represented in the form

$$\overrightarrow{U}(t,x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{ix\xi} \sum_{j,q=1}^{16} \frac{\Delta_{j,q}(\xi)}{\Delta(\xi)} e^{\lambda_q \xi} \overrightarrow{h}_q \psi_j(\xi) d\xi, \tag{3}$$

where $\Delta_{j,q}(\xi)$, $j,q=\{1,\ldots,4\}$, $\xi\neq 0$, – algebraic complement the standing element $j$ this poem and $q$ for this column of indicator $\Delta(\xi)$,

$$\mathrm{col}(\psi_1(\xi),\ldots,\psi_4(\xi))=\mathrm{col}(\tilde{\varphi}_1^1(\xi),\ldots,\tilde{\varphi}_1^4(\xi);\ldots;\tilde{\varphi}_4^1(\xi),\ldots,\tilde{\varphi}_4^4(\xi)),$$

i $\tilde{\varphi}_j^q(\xi)$ is a Fourier transformation of the funktion $\varphi_j^q(x)$, $j=\{1,...,4\}$, $q=\{1,...,4\}$. This result continues the research of work [2-9].

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