Supersymmetry, Supergravity and Non–Perturbative Dynamics of Gauge Theories

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Supersymmetry extends the Poincaré algebra [1] by introducing fermionic generators Q_α and $\bar{Q}_{\dot\alpha}$ satisfying

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu, \qquad \{Q_\alpha, Q_\beta\} = 0, \tag{1}$$

which enforce a correspondence between bosonic and fermionic degrees of freedom. This algebraic structure leads to cancellations in quantum corrections and constrains the form of effective actions.

In $\mathcal{N} = 1$ supersymmetric theories, the dynamics is encoded in the superspace action

$$S = \int d^4x\, d^4\theta\, K(\Phi_i, \Phi_i^\dagger) + \left[\int d^4x\, d^2\theta\, W(\Phi_i) + \text{h.c.}\right], \tag{2}$$

where K is the Kähler potential and W is the holomorphic superpotential. The scalar potential takes the universal form

$$V = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 + \frac{g^2}{2} \sum_a \left(\phi^\dagger T^a \phi\right)^2. \tag{3}$$

A remarkable realization of exact non–perturbative dynamics occurs in $\mathcal{N} = 2$ supersymmetric Yang–Mills theory, where the low-energy effective action is determined by a single holomorphic prepotential $\mathcal{F}(a)$. The physical variables are given by period integrals

$$a = \oint_\alpha \lambda_{\text{SW}}, \qquad a_D = \oint_\beta \lambda_{\text{SW}}, \tag{4}$$

with the Seiberg–Witten differential [2, 3]

$$\lambda_{\text{SW}} = \frac{x\, dx}{y}, \tag{5}$$

defined on the elliptic curve

$$y^2 = (x - e_1)(x - e_2)(x - e_3). \tag{6}$$

The BPS spectrum is exactly determined by the central charge

$$Z = n_e a + n_m a_D, \qquad M_{\text{BPS}} = |Z|, \tag{7}$$

and electric–magnetic duality acts as an $SL(2, \mathbb{Z})$ transformation

$$\begin{pmatrix} a_D \\ a \end{pmatrix} \to \begin{pmatrix} p & q \\ r & s \end{pmatrix} \begin{pmatrix} a_D \\ a \end{pmatrix}, \qquad ps - qr = 1. \tag{8}$$

Coupling to gravity promotes global supersymmetry to local supersymmetry, yielding $\mathcal{N} = 1$ supergravity with scalar potential

$$V = e^{K/M_{\text{Pl}}^2} \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - \frac{3}{M_{\text{Pl}}^2} |W|^2\right), \tag{9}$$

where

$$D_i W = \partial_i W + \frac{1}{M_{\text{Pl}}^2} (\partial_i K) W. \tag{10}$$

In string compactifications [4], moduli stabilisation can be achieved via the KKLT mechanism with superpotential

$$W = W_0 + A e^{-aT}, \tag{11}$$

leading to a scalar potential whose structure depends sensitively on higher-order corrections. In particular, α'^3 corrections modify the Kähler potential as

$$K = -2\ln\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right), \tag{12}$$

breaking the no-scale identity

$$K^{T\bar{T}}(\partial_T K)(\partial_{\bar{T}} K) = 3, \tag{13}$$

and generating distinct regimes of the potential, including metastable de Sitter vacua and runaway solutions.

This framework reveals a deep interplay between algebra, geometry, and quantum dynamics, where gauge theory observables are encoded in geometric data and vacuum structure is governed by holomorphic functions. The results provide a unified perspective on non–perturbative field theory, supergravity, and string compactifications, and shed light on fundamental questions concerning vacuum stability and the structure of the landscape.

References

[1] Julius Wess and Jonathan Bagger. *Supersymmetry and Supergravity.* Princeton: Princeton University Press, 1992.
[2] Nathan Seiberg and Edward Witten. Electric–magnetic duality, monopole condensation, and confinement in $\mathcal{N} = 2$ supersymmetric Yang–Mills theory. *Nuclear Physics B*, 426: 19–52, 1994.
[3] Nathan Seiberg and Edward Witten. Monopoles, duality and chiral symmetry breaking in $\mathcal{N} = 2$ supersymmetric QCD. *Nuclear Physics B*, 431: 484–550, 1994.
[4] Joseph Polchinski. *String Theory.* Cambridge: Cambridge University Press, 1998.