

On topological properties of weakly $(n - 1)$ -convex sets in low-dimensional Euclidean spaces

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We consider topological and geometric properties of generalized convex sets in the real n -dimensional Euclidean space \mathbb{R}^n , where $n = 2, 3$, known as weakly $(n - 1)$ -convex sets. An open set in \mathbb{R}^2 (resp. \mathbb{R}^3) is called weakly 1-convex (resp. weakly 2-convex) if through every boundary point of the set there passes a straight line (resp. a plane) not intersecting the given set. A closed set in \mathbb{R}^2 (resp. \mathbb{R}^3) is called weakly 1-convex (resp. weakly 2-convex) if it is approximated from the outside by a family of open weakly 1-convex (resp. weakly 2-convex) sets. A point in the complement of a set in \mathbb{R}^2 (resp. \mathbb{R}^3) is a 1-nonconvexity (resp. 2-nonconvexity) point of the set if every straight line (resp. every plane) passing through the point intersects the set.

It is shown that each connected component of the 1-nonconvexity-point set corresponding to a planar weakly 1-convex set is the interior of a convex polygon or the interior of a convex generalized polygon [1]. It is proved that the 2-nonconvexity-point set corresponding to an open weakly 2-convex set in \mathbb{R}^3 is itself open and weakly 2-convex [1], and that the non-empty interior of a solid closed weakly 2-convex set is weakly 2-convex [2]. A number of convexity properties of the 2-nonconvexity-point sets corresponding to weakly 2-convex sets in \mathbb{R}^3 are also investigated. In particular, it is shown that for any convex polyhedron there exists an open weakly 2-convex set such that its 2-nonconvexity-point set coincides with the polyhedron interior; however, there exists an open weakly 2-convex set such that its open bounded convex 2-nonconvexity-point set differs from the interior of a convex polyhedron [2]. Nevertheless, not every open convex set can occur as a connected component of the 2-nonconvexity-point set corresponding to some open weakly 2-convex set [1].

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REFERENCES

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