On convex bodies with planar projections having rotational symmetries

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Let $n \geq 3$ and let $K \subset \mathbb{R}^n$ be a convex body. For a two-dimensional linear subspace $P \subset \mathbb{R}^n$, let $K|P$ be the orthogonal projection of K onto P. We prove that if for every two-dimensional subspace P, the planar convex body $K|P$ has q-fold rotational symmetry up to translation, then for $q \geq 4$, this forces K to be a ball. The case $q = 3$ is exceptional: non-spherical examples exist.