

On the rational points and zeta functions of the weighted projective spaces

Sajad Salami

Rio de Janeiro State University, Rio de Janeiro, Brazil

E-mail: gnipun@ttu.edu

Weighted projective spaces ($\mathbb{P}_{\mathbf{w}}^n$) offer a rich arithmetic landscape, yet calculating their \mathbb{F}_q -rational points often presents unique challenges compared to standard projective space. I will discuss our recent work [1] with Tony Shaska (2025). In this presentation, we clarify the geometry of these spaces by establishing the equivalence of three distinct notions of rational points. Then, I demonstrate how to derive explicit combinatorial formulas for point enumeration using Burnside's Lemma and gcd computations, while providing closed-form expressions for both the singular and smooth loci. Furthermore, we will explore the resulting zeta functions $Z(\mathbb{P}_{\mathbf{w}}^n, t)$, showing how they admit a canonical multiplicative decomposition that mirrors the space's internal stratification.

REFERENCES

- [1] Sajad Salami and Tanush Shaska. Rational points in weighted projective spaces over finite fields, 2025. [doi:10.48550/arXiv.2511.12812](https://doi.org/10.48550/arXiv.2511.12812).