Basic aspects of some special geometry of tangent bundle for spaces of affine connection

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We consider a space of affine connection A^n $(n > 2)$ of the class of smooth $C^r(r > 1)$ with the object of affine connection Γ that is determined by components $\Gamma^h_{ij}(x^1, x^2, \ldots, x^n),\ h, i, j = \overline{1, n}$ according to every local coordinate system $(x^i),\ i = \overline{1, n}$ [1].

Some special broadened affine connection $\tilde{\Gamma}$ is built. It has the components

$$\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots, y^n), \quad h, i, j = \overline{1, n},$$

according to every local coordinate system,

$$\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, \ldots, y^n) = \Gamma^h_{ij}(x^1, x^2, \ldots, x^n) + T^h_{(ij)\alpha} y^\alpha, \quad h, i, j, \alpha = \overline{1, n}.$$

Here

$$T^h_{ij\alpha} = \delta^h_i R_{j\alpha} - R^h_{ij\alpha},$$

δ^h_i are the Kronecker delta symbols, $R_{j\alpha}$ are the Ricci symbols, $R^h_{ij\alpha}$ are the Riemann symbols of the space A^n, parentheses denote symmetrization of the included indices without division. In contrast to components of the object Γ, components of the object $\tilde{\Gamma}$ depend on $2n$ variables. It is natural to consider them in the mentioned way to be defined in the space of the tangent bundle $T(A^n)$. At the same time, proceeding from the number of components $\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots, y^n)$, the broadened connection $\tilde{\Gamma}$ may be considered as a connection in A^n that depends not only on the coordinates of the current point but also on the components of the tangent vector in it. This means that the connection $\tilde{\Gamma}$ is in some sense similar to connections of Cartan and Berwald of Finsler geometry.

On the base of the connection $\tilde{\Gamma}$, by the help of the operation of the type of full lift, the connection $c\tilde{\Gamma}$ is built.

Its components $c\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots, y^n)$ are the following:

$$c\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots, y^n) = \tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots$$

$$c\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots, y^n) = \frac{\partial\tilde{\Gamma}^{h-n}_{ij}(x^1, x^2, \ldots, x^n; y^1, y}{\partial x^\alpha}$$

$$h = \overline{n+1, 2n},\ i, j, \alpha = \overline{1, n};$$

$$c\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots, y^n) = \tilde{\Gamma}^{h-n}_{ij-n}(x^1, x^2, \ldots, x^n; y^1, y^2$$

$$c\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots, y^n) = \tilde{\Gamma}^{h-n}_{i-nj}(x^1, x^2, \ldots, x^n; y^1, y^2$$

$$c\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots, y^n) = 0,\ i = \overline{n+1, 2n},\ h, j = 1,$$

$$c\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots, y^n) = 0,\ i, j = \overline{n+1, 2n},\ h = 1,$$

$$c\tilde{\Gamma}^h_{ij}(x^1, x^2, \ldots, x^n; y^1, y^2, \ldots, y^n) = 0,\ i = \overline{n+1, 2n},\ h, i, j =$$

By the help of the connection $c\tilde{\Gamma}$ the covariant differentiation is introduced in $T(A^n)$, by the traditional way the concept of geodesic line is defined. Interdependences between the concept of a geodesic line in the space A^n according to the connection Γ, the concept of a geodesic line in this space A^n according to the connection $\tilde{\Gamma}$ and the concept of a geodesic line in the space $T(A^n)$ according to the connection $\tilde{\Gamma}$ are investigated.

References

[1] Josef Mikeš, Alena Vanžurová, and Irena Hinterleitner. *Differential Geometry of Special Mappings*. Palacký University Press, Olomouc, Czech Republic, 2nd edition, 2019. doi:10.5507/prf.19.24455358.