Coding of flows with singularities on the boundary of the two-dimensional disk

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We investigate topological properties, possible structures, classifications and coding of flows on a two-dimensional disk, which have a finite number of separatrices and all singular points of the flow lie on the boundary of the disk. Trees with selected leaves are used to classify such flows. We construct the code for a flow with one singular point on the boundary.

Definition 1. A distinguished graph of a flow is a planar graph obtained from a bivalent graph by deleting the outer vertex and the half-edges incident to it, and equipped with the following additional data:

(1) all edges are oriented;
(2) some leaves of the graph are distinguished;
(3) at each vertex of the graph, a sector is distinguished; this sector can be specified by an ordered pair of adjacent half-edges incident to the vertex: the first half-edge corresponds to the separatrix entering the boundary of the sector, and the second to the separatrix leaving it.

Theorem 2. *Let a flow be given on the 2-disk, having a finite number of singular points and separatrices, and suppose that all singular points lie on the boundary of the 2-disk. Then the distinguished graph of such a flow is a tree.*

Theorem 3. *The flows on the 2-disk that have a finite number of singular points and separatrices, with all singular points lying on the boundary of the 2-disk, are topologically equivalent with preservation of orientation if and only if their distinguished graphs are equivalent.*

Definition 4. A 1-flow on the 2-disk D^2 is a flow that has a finite number of separatrices, exactly one singular point, and this point lies on the boundary of the 2-disk ∂D^2.

Definition 5. The distinguished graph of a 1-flow is a two-colored rooted tree with labels on its edges and with ordering of the lower edges at each vertex.

Two distinguished graphs are called equivalent if there exists an isomorphism between them that preserves the colors, labels, and orderings of the lower edges, and maps the root to the root.

Corollary 6. *Two 1-flows on the 2-disk are topologically trajectory equivalent if and only if their distinguished graphs are equivalent.*

In the article, we introduce the notion of a 1-code, for which the following statements hold:

Theorem 7. *Two* 1*-flows are topologically trajectory equivalent if and only if their codes are identical* [1].

References

[1] Olexandr Prishlyak and Serhii Stas. Coding of flows with singularities on the boundary of the two-dimensional disk. *Nonlinear Oscillations*, 28(1):114–126, 2025. (in Ukrainian). doi:10.48550/arXiv.2304.00751.