Representations of the p-adic rotation group towards p-adic quantum computing

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We investigate the structure of the p-adic rotation group $SO(3)_p$, and initiate a program aimed at classifying its finite-dimensional irreducible projective unitary representations. Our approach relies on the profinite nature of $SO(3)_p$, together with its Haar measure. These representations can be interpreted as a theory of p-adic angular momentum and spin, where the p-adic qubit arises as a two-dimensional representation. Indeed, p-adic numbers find fruitful applications in p-adic formulations of quantum mechanics, and in dynamical systems, where their ultrametric topology naturally models hierarchical and fractal-like structures.

We describe the foundations of our program, starting from the main features of $SO(3)_p$ (in parallel to its real counterpart), such as a p-adic analogue of the Cardano (aka nautical) angles decomposition [3], and a description of rotations in terms of p-adic quaternions [1]. We characterise the profinite group $SO(3)_p$ as an inverse limit of the inverse family of groups $SO(3)_p \bmod p^k$ [2]. We show that all finite-dimensional projective unitary representations of $SO(3)_p$ factorise on some $SO(3)_p \bmod p^k$, and we find explicit p-adic qubit representations for every prime p. Interestingly, there are several p-adic qubit representations for $p > 3$ [6].

Then, it is natural to compose systems of multiple p-adic qubits, through the tensor product of their representations. We solve the Clebsch–Gordan problem for systems of two p-adic qubits from $SO(3)_p \bmod p$, revealing that the coupled bases decompose into singlet and doublet states [4]. We further study entanglement arising from those stable subsystems: every singlet, doublet (and triplet) can be given by maximally entangled Bell states. However, except for the singlets, the projectors onto doublets (and triplets) are separable quantum states [5].

Lastly, we propose a circuit model of quantum computation where logic gates are driven by the actions of $SO(3)_p$ [4]. For $p = 3$, we construct a set of gates from four-dimensional irreducible representations of $SO(3)_3 \bmod 3$, that we prove to be universal for quantum computation [5].

References

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