Exact solutions of nonlinear dispersive evolution equations

Olena Vaneeva
Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine
E-mail: vaneeva@imath.kiev.ua

Alexander Zhalij
Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine
E-mail: zhaliy@imath.kiev.ua

Olena Magda
National Technical University of Ukraine "Kyiv Polytechnic Institute", Ukraine
E-mail: o.v.magda@gmail.com

Oksana Brahinets
Petro Mohyla Black Sea National University, Mykolaiv, Ukraine
E-mail: oksana.brahinets@gmail.com

Many model equations for various wave processes can be reduced to nonlinear dispersive evolution equations such as the classical Korteweg–de Vries equation, the modified Korteweg–de Vries equation, and the Kawahara equation, as well as their generalizations, which often involve variable coefficients. This explains the great interest of researchers in finding new and applying already known methods for constructing exact solutions to such generalized equations. At the same time, most of the proposed methods lead to equivalent forms of already known solutions, primarily because the equivalence of models and corresponding solutions is not systematically investigated. This widespread oversight in finding solutions to differential equations was thoroughly analyzed in [3]. Conversely, the application of equivalence transformations serves as an effective tool for finding exact solutions, solving group classification problems, and investigating the integrability of nonlinear equations of mathematical physics with variable coefficients [3, 2, 7]. So, the equivalence method can be proposed as a method of choice for a wide range of such problems.

In this talk, we discuss the use of equivalence transformations for constructing exact solutions of nonlinear dispersive evolution equations of KdV type with variable coefficients. The respective results used as illustrative examples have been published, in particular, in [2, 1, 5, 6, 4].

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