Supersymmetry, Supergravity and Non-Perturbative Dynamics of Gauge Theories

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Outline

Motivation

- Supersymmetry extends Poincaré algebra with fermionic generators:

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^{\mu}_{\alpha\dot{\alpha}} P_\mu$$

- Cancellation of bosonic/fermionic loops $\rightarrow$ UV finiteness.
- Seiberg–Witten solution: exact non-perturbative dynamics encoded in geometry.
- Local SUSY $\rightarrow$ supergravity $\rightarrow$ couples geometry of scalar manifold to spacetime.
- String compactifications give geometric origin to K, W, f_{ab}.
- **Focus:** Moduli stabilisation & de Sitter vacua (Section 6).

$\mathcal{N} = 2$ SU(2) Yang–Mills: Coulomb Branch

- Gauge invariant coordinate: $u = \langle \mathrm{Tr}\, \phi^2 \rangle$.
- Seiberg–Witten curve (elliptic):

$$y^2 = (x - e_1)(x - e_2)(x - e_3), \qquad \lambda_{SW} = \frac{x\, dx}{y}$$

- Periods:

$$a = \oint_\alpha \lambda_{SW}, \quad a_D = \oint_\beta \lambda_{SW} = \frac{\partial \mathcal{F}}{\partial a}$$

- Prepotential:

$$\mathcal{F}(a) = \frac{i}{2\pi} a^2 \ln \frac{a^2}{\Lambda^2} + \sum_{k=1}^{\infty} \mathcal{F}_k \left(\frac{\Lambda}{a} \right)^{4k} a^2$$

- BPS mass: $M = |n_e a + n_m a_D|$.
- Monodromy $\in SL(2, \mathbb{Z}) \to$ electromagnetic duality.

$\mathcal{N} = 1$ Supergravity: Master Action

- Global SUSY: $S = \int d^4x d^4\theta \, K + \left(\int d^4x d^2\theta \, W + \text{h.c.} \right)$.
- Localisation: $\epsilon_\alpha \to \epsilon_\alpha(x) \to$ gravitino $\psi_{\mu\alpha}$ and graviton $g_{\mu\nu}$.
- Curved superspace: $d^4x d^4\theta \to d^4x d^4\theta \, E$, $d^4x d^2\theta \to d^4x d^2\theta \, \mathcal{E}$.
- Kähler function:

$$\mathcal{G} = \frac{K}{M_{\text{Pl}}^2} + \ln \frac{|W|^2}{M_{\text{Pl}}^6}$$

- Scalar potential:

$$V = M_{\text{Pl}}^4 e^{\mathcal{G}} \left(g^{i\bar{j}} \mathcal{G}_i \mathcal{G}_{\bar{j}} - 3 \right) = e^{K/M_{\text{Pl}}^2} \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - \frac{3}{M_{\text{Pl}}^2} |W|^2 \right)$$

- Negative term enables de Sitter uplift.

D-Branes and Geometric Engineering

- D3-branes: $\mathcal{N} = 4$ SYM, $U(N)$.
- Orbifolds / fluxes $\to \mathcal{N} = 2, 1$.
- Flux superpotential (GVW):

$$W_{\text{flux}} = \int_{X_6} G_3 \wedge \Omega, \qquad G_3 = F_3 - \tau H_3$$

- Kähler potential for moduli:

$$K = -\ln\left(i \int_{X_6} \Omega \wedge \bar{\Omega}\right) - 2\ln \mathcal{V} + K_{\text{matter}}$$

- Seiberg–Witten curve = brane separation geometry.
- AdS/CFT: $\mathcal{N} = 4$ SYM $\leftrightarrow$ IIB supergravity on $AdS_5 \times S^5$.

The KKLT Construction

- Type IIB on Calabi–Yau orientifold: flux stabilises complex structure and dilaton.
- Kähler modulus $T = \sigma + i\theta$ remains flat (no-scale structure):

$$K_{\rm cl} = -3\ln(2\sigma), \qquad K^{T\bar{T}}(\partial_T K)(\partial_{\bar{T}} K) = 3$$

- Non-perturbative superpotential:

$$W = W_0 + Ae^{-aT}$$

  (W_0 from fluxes, Ae^{-aT} from gaugino condensation / ED3-instantons)
- SUSY AdS minimum at $D_T W = 0$:

$$V_{\rm AdS} = -3e^{K_0}|W_0|^2 < 0$$

- Uplift by $\overline{\rm D3}$-brane: $V_{\rm total} = V(\sigma) + \frac{D}{\sigma^3}$, $D > 0$.

α'^3 Corrections to Kähler Potential

- Leading perturbative correction (Becker et al.):

$$\delta K^{(\alpha'^3)} = -\frac{\hat{\xi}}{2\mathcal{V}}, \qquad \hat{\xi} = -\frac{\chi(X_6)}{2(2\pi)^3}\frac{\zeta(3)}{g_s^{3/2}} > 0$$

- Corrected Kähler potential (single modulus):

$$K = -3\ln(2\sigma) - \frac{\hat{\xi}}{16\sigma^{3/2}}$$

- Breaks no-scale identity:

$$K^{T\bar{T}}(\partial_T K)(\partial_{\bar{T}} K) = 3 - \frac{9\hat{\xi}}{8\mathcal{V}} + \cdots$$

- Activates the $-3|W|^2$ term in V even before non-perturbative effects.

Scalar Potential with α'^3 Correction

- After axion minimisation ($\cos(a\theta_0 + \phi) = +1$):

$$V(\sigma) = \underbrace{\frac{a^2|A|^2 e^{-2a\sigma}}{2\sigma}}_{V_{\rm np}^{(1)}} - \underbrace{\frac{a|A||W_0|e^{-a\sigma}}{2\sigma^2}}_{V_{\rm np}^{(2)}} + \underbrace{\frac{3\hat{\xi}|W_0|^2}{8\sigma^3}}_{V_\xi}$$

- $V_\xi > 0$ competes with the negative $V_{\rm np}^{(2)}$.
- Shift of the AdS minimum toward larger volume.

Three Regimes of the Potential

Regime	Condition	Physics
I	$\hat{\xi} = 0$	Classical KKLT: AdS min at σ_0^{cl}
II	$0 < \hat{\xi} < \hat{\xi}_c$	Corrected KKLT: AdS min at $\sigma_0 > \sigma_0^{\text{cl}}$
III	$\hat{\xi} > \hat{\xi}_c$	No minimum (runaway) $\to$ LVS or decompactification

Critical parameter:

$$\hat{\xi}_c \approx \frac{(a\sigma_0^{\text{cl}})^{3/2}}{3\sqrt{2}} \frac{|A|e^{-a\sigma_0^{\text{cl}}}}{|W_0|}$$

Implications for de Sitter Vacua

- For typical KKLT parameters $|W_0| \ll |A|e^{-a\sigma_0^{\text{cl}}} \to \hat{\xi}_c \gg 1$ (in Planck units).
- Hence α'^3 correction does *not* destroy the AdS minimum; it merely shifts it.
- Uplift to dS remains possible if $D \approx |V_{\text{AdS}}|\sigma_0^3$ and $\delta\sigma/\sigma_0 \sim 1/(a\sigma_0) \ll 1$.
- However, near $\hat{\xi} \lesssim \hat{\xi}_c$ the potential becomes very flat $\to$ tension with de Sitter swampland conjecture:

$$|\nabla V| \geq c\, V, \qquad c \sim \mathcal{O}(1)$$

- Our three-regime map provides a concrete arena to test swampland bounds.

Open Questions & Future Directions

- **Gauge theory:** Higher-rank groups, matter hypermultiplets, AGT correspondence.
- **Supergravity:** Non-perturbative corrections to K and W, Kähler curvature effects.
- **String realisations:** Geometric engineering beyond $SU(2)$, holographic confinement.
- **Moduli stabilisation:** Multi-modulus generalisations, higher α' and g_s corrections, alternative uplifts (F-term, D-term, T-branes).
- **Swampland:** Quantitative comparison of $|\nabla V|/V$ and Hessian eigenvalues with refined conjectures.
- **Cosmology:** Kähler moduli inflation, axion natural inflation, brane inflation – connecting phase diagram to (n_s, r).

Conclusion

"The vacuum structure of supersymmetric theories is encoded in the geometry of holomorphic objects."

- Seiberg–Witten: $\mathcal{N} = 2$ dynamics elliptic curve periods.
- $\mathcal{N} = 1$ supergravity: K, W, f_{ab} fix all sectors.
- String theory: K, W, f_{ab} descend from Calabi–Yau geometry and fluxes.
- KKLT $+ \alpha'^3$ corrections $\rightarrow$ three distinct regimes; critical $\hat{\xi}_c$ separates controlled dS vacua from runaway.
- The interplay between perturbative corrections, non-perturbative effects and swampland constraints remains a central challenge for string phenomenology.

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