Structure of gradient flows with a degenerate node on the sphere with holes

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Contents

Morse flows

Let X be a smooth vector field with a finite number of singular points on a compact smooth surface M so that it generates a flow g on M.
Suppose that all the vectors of X defined on ∂M are tangent to ∂M.

Morse flows

Definition

A vector field X is called a Morse vector field if next conditions hold true:

1) all its singular points are non-degenerate;
2) the set of its non-wandering points is equal to the set of its singular points, and $\mathrm{Int}(M)$ does not contain any saddle connections.

Definition

A flow g on a compact surface M is called a Morse flow if it is generated by a Morse vector field.

HS- and *HN*-flows

Definition

A vector field X is called an $HS_{\pm}$-vector field if:

1) it contains one degenerate saddle hs on the boundary but all its other singular points are non-degenerate;
2) it can be expressed through the formula

$$V(x,y) = \{\pm x, \mp y^2\},\ x, y \in [-1,1] \times [0,1]$$

in local coordinates in the neighbourhood of hs;
3) the set of its non-wandering points is equal to the set of its singular points, and $\mathrm{Int}(M)$ does not contain any saddle connections.

HS- and HN-flows

Definition

A flow g on a compact surface M is called an HS_+-flow (HS_--flow) if it is generated by an HS_+-vector field (HS_--vector field).

HS- and *HN*-flows

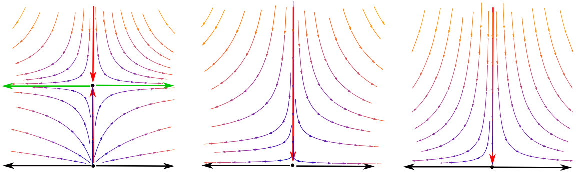

Fig.: HS_+ bifurcation

HS- and *HN*-flows

Definition

A vector field X is called an $HN_{\pm}$-vector field if:

1) it contains one degenerate source (sink) hn on the boundary but all its other singular points are non-degenerate;
2) it can be expressed through the formula

$$V(x, y) = \{\pm x, \pm y^2\},\ x, y \in [-1, 1] \times [0, 1]$$

   in local coordinates in the neighbourhood of hn;
3) the set of its non-wandering points is equal to the set of its singular points, and $\mathrm{Int}(M)$ does not contain any saddle connections.

HS- and HN-flows

Definition

A flow g on a compact surface M is called an HN_+-flow (HN_--flow) if it is generated by an HN_+-vector field (HN_--vector field).

HS- and *HN*-flows

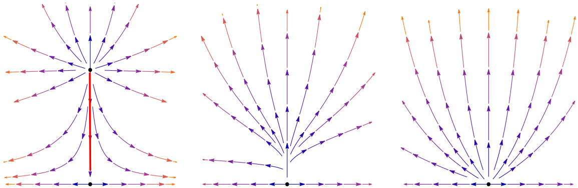

Fig.: HN_+ bifurcation

Distinguishing graph

The problem of construction a complete topological graph for Morse flows before the moment of bifurcation was researched. It is a distinguishing graph which generalizes a notion of a chord diagram from the previous work.
Apart from that, a code defining each distinguishing graph and thus the Morse flow itself was constructed.

Distinguishing graph

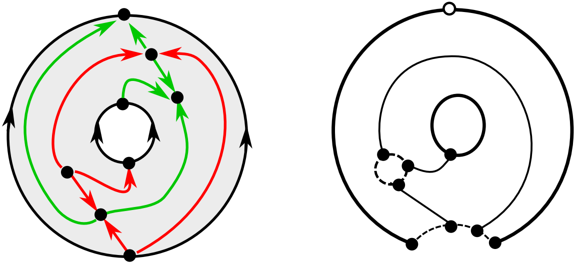

Fig.: An example of a distinguishing graph for a Morse flow on the sphere with 2 holes.
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Distinguishing graph

Theorem

HS-flows (*HN*-flows) are topologically equivalent if and only if the appropriate distinguishing graphs are isomorphic so that the isomorphism preserves the colours of all the vertices and the edges.

Main result

There are 4 HS_+- and 6 HN_+-flows with 5 singular points on two-dimensional disk (D^2).
There are 13 HS_+- and 24 HN_+-flows with 6 singular points on two-dimensional disk (D^2).