Invariant idempotent measures

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The notion of invariant (self-similar) measure for an iterated function system (IFS) of contractions on a complete metric space is introduced in [1]. The existence of invariant measures is proved by using the Banach contraction principle for suitable metrization of the set of probability measures on a metric space. The invariant measures impose an additional structure on the invariant set for the given IFS.

The aim of the talk is to introduce the invariant idempotent measures for given IFS. Recall that an idempotent measure on a compact Hausdorff space X is a functional $\mu\colon C(X) \to \mathbb{R}$ that preserves constants, the maximum operation (usually denoted by $\oplus$) and is weakly additive (i.e., preserves sums in which at least one summand is a constant function; we use $\odot$ for these sums) [3]. Given an arbitrary metric space X, we denote by $I(X)$ the set of idempotent measures of compact supports on X. In the case of idempotent measure, we use the weak* convergence for proving the existence of invariant element. This approach seems to be fairly general and we anticipate new results in this direction. Note also that the invariant idempotent measures on ultrametric spaces are introduced and investigated in [2].

Let X be a complete metric space and let $f_1, f_2, \dots, f_n$ be an IFS on X. We assume that all f_i are contractions. Let also $\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}$ be such that $\bigoplus_{i=1}^n \alpha_i = 0$.

By $\exp X$ we denote the hyperspace of the space X endowed with the Hausdorff metric. Let Ψ_0 denote the identity map of $\exp X$ and, for $i > 0$, define $\Psi_i\colon \exp X \to \exp X$ inductively:

$$\Psi_i(A) = \bigcup_{j=1}^{n} f_{j-1}(\Psi_{i-1}(A)).$$

Let $\Phi_0\colon I(X) \to I(X)$ be the identity map. For $i > 0$, define $\Phi_i\colon I(X) \to I(X)$ inductively: $\Phi_i(\mu) = \bigoplus_{j=1}^n \alpha_j \odot I(f_j)(\Phi_{i-1}(\mu))$. Thus, $\Phi_i = \Phi_1\Phi_1 \cdots \Phi_1$ (i times). It is easy to check that the maps Φ_i are well-defined.

In this case, we say that $\mu \in I(X)$ is an *invariant idempotent measure* if $\Phi_i(\mu) = \mu$ for every $i = 0, 1, \dots$ (equivalently, $\Phi_1(\mu) = \mu$).

Theorem 1. *There exists a unique invariant idempotent measure for the IFS $f_1, f_2, \dots, f_n$ and $\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}$ with $\bigoplus_{i=1}^n \alpha_i = 0$. This invariant measure is the limit of the sequence $(\Phi_i(\mu))_{i=1}^\infty$, for arbitrary $\mu \in I(X)$.*

Rerefences

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[2] N. Mazurenko, M. Zarichnyi, Idempotent ultrametric fractals. *Visnyk of the Lviv Univ. Series Mech. ath.*, Issue 79 : 111–118, 2014.
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