Weak and strong nilpotentizability of vector distributions

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1. Cartan prolongation of vector distributions, classical and generalized.

Examples. Local polynomial normal forms.

Goursat flags as the outcomes of series of classical Cartan prolongations.

Special multi-flags as the outcomes of series of generalized Cartan prolongations.

Rich trees of singularities emerging in these constructions: from the length three (so-called *modèle exceptionnel* of Kumpera & Ruiz of 1978) onwards in the case of Goursat flags, and from the length two in the case of special multi-flags.

2. Nilpotent Approximation procedure. Examples.

Definition of strong nilpotentizability. Examples of not strongly nilpotent points in the Goursat Monster Tower (GMT) from the length four onwards.

Definition of weak nilpotentizability (= the classical nilpotentizability in geometric control theory).

Strong implies weak. Are both equivalent?!

3. Theorem (2000) that the entirety of the GMT is weakly nilpotent.

Formulas for the nilpotency orders (or: steps) of the emerging nilpotent Lie algebras.

Open question concerning strong nilpotentizability in the GMT.

Theorem (2004) on the weak nilpotentizability of the special multi-flags.

The nilpotency orders of the respective emerging nilpotent Lie algebras.

Open questions in the Special Multi-Flags Monsters.

Rerefences

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[3] Richard Montgomery & Michail Zhitomirskii. Points and Curves in the Monster Tower. *Memoirs of the American Mathematical Society* **956** (2010).
[4] Piotr Mormul. Goursat distributions not strongly nilpotent in dimensions not exceeding seven. Volume **281** (2003) of *Lecture Notes in Control and Information Sciences*, 249 – 261.
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[6] Piotr Mormul. Singularity classes of special 2-flags. *SIGMA* **5** (2009), 102, 22 pages (electronic).