Modernized Dijkstra's algorithm and mathematical model for geographic information system

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For modeling of a domain of the Earth there are usually used a cartographic projection. The goal of our investigation is method of finding of optimal path between adjacent control points that are in some vertices of graph. The tusk of line orienteering is to find optimal path between adjacent control points in some vertices of graph. The goal of our investigation is to construct such method of finding path through all control points in determed sequence. We can choose optimal path between every pair of control points (c.p.).

For this goal we develop modified Dijkstra's algorithm that has complexity $O(\frac{V^2}{4}+\frac{VlnV}{4})$ instead of usual complexity $O(V^2+V\log_2 V)$. Also our algorithm permits a parallel realizing. As a result of a parallel implementation complexity of algorithm decrease in $|4(|V_o|-1)|$ times, where $|V_o|$ is number of c.p. Coefficient of speedup of an the algorithm of parallel search is

$$\kappa_1 = \frac{(V^2+V\log_2 V)\cdot|4(|V_o|-1)|}{V^2+V\log_2 V} = \frac{|4(|V_o|-1)|}{1}.$$

Let n is number of nods. And coefficient of extension by parallelling

$$\kappa_2 = \frac{\kappa_1}{n}$$

We propouse to find an optimal path between two vertices using a opposition search method. This method are based on parallel opposition search of shortest path by Dijkstra's algorithm from two vertices A and B which are c.p. on oriented graph. First vertex with minimal distance to both vertices A and B is enclose vertex of a path from A to B.

Model for a sequential search of path was constructed. Let e_{ij} is an edge between vertices v_i and v_j that has length d_{ij}. We consider a resistance r_{ij} of e_{ij} and determine new length of labeled graph edge as $\widetilde{d_{ij}} = r_{ij}d_{ij}$. The model for n competitor of sport orienteering has weighted sum objective function

$$\mathbb{F}(v_0, v_n, k) = \sum_{s=i}^{j} d_{ij} r_{ij} s_{ij}^{-1} \to \min,$$

with inequality restriction $r_{ij} < R(k)$, where $R(k)$ is value of a critical load of k-th competitor of orienteering search, s_{ij} – speed of moving by e_{ij} [1, 2].

Let S – set of all c.p. If we can chose set S, then we use method of mask dynamic of search for solution of the problem

$$\min_{2\le j\le n} \left(dp\left(\{1,2,\ldots,n\},j\right) + m\left[j,i\right]\right),$$

$$dp\left(S,i\right) = \min_{j\in S\setminus\{i\}} \left(dp\left(S\setminus[i],j\right) + m\left[j,i\right]\right).$$

Where $dp\left(S,i\right)$ is shortest path, which starts in v_1 and going by all vertices from $S\setminus\{v_i\}$, ending up in vertex v_j.

Rerefences

[1] R. V. Skuratovskii *Generilized Euler's constant.* Vol. 10, *Mathematical visnuk of NTSh.* pp. 163-168., 2013.
[2] R. V. Skuratovskii *Modernized Pohlig-Hellman and Shanks algorithm.* Vol. 1 *Visnuk of KNU. Cybernetics.* pp. 56., 2015.