almost and weakly neighborhood star-Menger

Lakehal Rachid [1], Lakehal Ibrahim [2]
[1] Department of mathematics, University of Bejaia, Algeria
[2] Department of mathematics, University Mohamed Elbachir Elibrahimi BBA, Algeria
r.lakehal@univ-boumerdes.dz, ibrahimlak1991@gmail.com

We introduce and study some types of star-selection principles (almost and weakly neighborhood star-Menger). We establish some properties of these selection principles and their relations with other selection properties of topological spaces. The behavior of these classes of spaces under certain kinds of mappings is also considered.

We give firstly definitions of notions which are used in this work.

Throughout of the work $\mathbb{N}$ and $\mathbb{R}$ denote the set of positive integers and the set of real numbers. Let X be a topological space, $\mathcal{U}$ a collection of subsets of X, and $A \subset X$. Then the set $\mathrm{St}(A,\mathcal{U}) := \bigcup\{P \in \mathcal{U} \ : P \cap A \neq \emptyset\}$ is called the star of A with respect to $\mathcal{U}$. As usual, we write $\mathrm{St}(x,\mathcal{U})$ instead of $\mathrm{St}(\{x\},\mathcal{U})$(see [2]) . Let $\mathcal{A}$ and $\mathcal{B}$ be collections of covers of a space X. Then the symbol $\mathsf{S}_1(\mathcal{A},\mathcal{B})$ (see [2]) denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(U_n : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $U_n \in \mathcal{U}_n$ and $\{U_n : n \in \mathbb{N}\}$ is an element of $\mathcal{B}$. The symbol $\mathsf{S}_{\mathsf{fin}}(\mathcal{A},\mathcal{B})$ (see [2]) denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(\mathcal{V}_n \ : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ is an element of $\mathcal{B}$.

Kočinac (see [2]) introduced star selection hypothesis similar to the previous ones. Let $\mathcal{A}$ and $\mathcal{B}$ be collections of covers of a space X. Then:

(**A**) The symbol $\mathsf{S}_{\mathsf{fin}}{}^*(\mathcal{A},\mathcal{B})$ denotes the selection hypothesis that for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there exists a sequence $(\mathcal{V}_n : n \in \mathbb{N})$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and $\bigcup_{n\in\mathbb{N}}\{\mathrm{St}(V,\mathcal{U}_n) \ : \ V \in \mathcal{V}_n)\}$ is an element of $\mathcal{B}$.

(**B**) The symbol $\mathsf{SS}^*_{\mathrm{comp}}(\mathcal{A},\mathcal{B})$ $(SS^*_{fin}(A,B))$ denotes the selection hypothesis that for each sequence $(\mathcal{U}_n \ : n \in \mathbb{N})$ of elements of A there exists a sequence $(K_n : n \in \mathbb{N})$ of compact (resp., finite) subsets of X such that $\{St(K_n,\mathcal{U}_n) : n \in \mathbb{N}\} \in B$.

Let $\mathcal{O}$ denote the collection of all open covers of a space X.

Definition 1. [3] A space X is said to be *star-Menger* [resp, *star-Rothberger*] if it satisfies the selection hypothesis $\mathrm{S}_{\mathrm{fin}}{}^*(\mathcal{O},\mathcal{O})$ [resp, $\mathsf{S}_1{}^*(\mathcal{O},\mathcal{O})$]

The following three generalizations of star selection properties have been introduced (in a general form and under different names).

Definition 2. [1] A space X is said to be *neighbourhood star-Menger* (NSM) if for every sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X, one can choose finite $(F_n \subset X,\ n \in \mathbb{N})$, so that for every open $O_n \supset F_n$, $n \in \mathbb{N}$, we have $\bigcup_n\{\mathrm{St}(O_n,\mathcal{U}_n) : n \in \mathbb{N}\} = X$.

So In this work we introduce weaker versions of neighbourhood star-Menger spaces (almost and weakly neighborhood star-Menger).

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