On Closedness of Right(Left) Normal Bands and Left(Right) Quasinormal Bands

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It is known that all subvarieties of variety of all semigroups are not absolutely closed. So, it is obvious to study which subvarieties of variety of all semigroups are closed in itself or closed in containing subvarieties. In this direction, Scheiblich [6] has shown that the variety of all normal bands is closed while Alam and Khan [2, 3, 4] have shown that the variety of left [right] regular bands, left [right] quasinormal bands and left [right] seminormal bands are closed. In [1], Ahanger and Shah have shown that the variety of left[right] regular bands is closed in the variety of all bands.

Higgins [5, Chapter 4] has shown that variety of right normal bands is not absolutely closed. So, it is worthy of attention to find out the subvarieties of varieties of all semigroups in which the varieties of right[left] normal bands and left(right) quasinormal bands are closed. In this paper, we have shown that the varieties of right [left] normal bands and left [right] quasinormal bands are closed in the varieties of semigroups defined by the identities $axy = xa^ny$ $[axy = ay^nx]$, $axy = x^nay$ $[axy = ayx^n]$ $(n > 1)$; and $axy = ax^nay$ $[axy = ayx^ny]$ $(n > 1)$, $axy = a^nxa^ry$ $[axy = ay^rxy^n]$ $(n, r \in \mathbf{N})$, respectively.

Some basic definitions are as follows:

Definition 1. A band S is said to be a left[right] normal band if S satisfies the identity $axy = ayx[axy = xay]$ for all $a, x, y \in S$.

Definition 2. A band S is said to be a normal band if S satisfies the identity $axya = ayxa$.

Definition 3. A band S is said to be a left[right] regular band if S satisfies the identity $ax = axa[xa = axa]$ for all $a, x \in S$.

Definition 4. A band S is said to be a left[right] quasinormal band if S satisfies the identity $axy = axay[axy = ayxy]$ for all $a, x, y \in S$.

Definition 5. A band S is said to be a left[right] seminormal band if S satisfies the identity $axy = axyay[axy = ayaxy]$ for all $a, x, y \in S$.

Finally, we prove the following Theorems:

Theorem 1. *The variety $\mathcal{V} = [axy = xa^ny]$ $(n > 1)$ of semigroups; i.e., the class of all semigroups satisfying the identity $axy = xa^ny$ is closed.*

Theorem 2. *The variety $\mathcal{V} = [axy = x^nay]$ $(n > 1)$ of semigroups; i.e., the class of all semigroups satisfying the identity $axy = x^nay$ is closed.*

Theorem 3. *The variety $\mathcal{V} = [axy = ax^nay]$ $(n > 1)$ of semigroups; i.e., the class of all semigroups satisfying the identity $axy = ax^nay$ is closed.*

Theorem 4. *The variety $\mathcal{V} = [axy = a^nxa^ry]$ $(n, r \in \mathbf{N})$ of semigroups; i.e., the class of all semigroups satisfying the identity $axy = a^nxa^ry$ is closed.*

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